



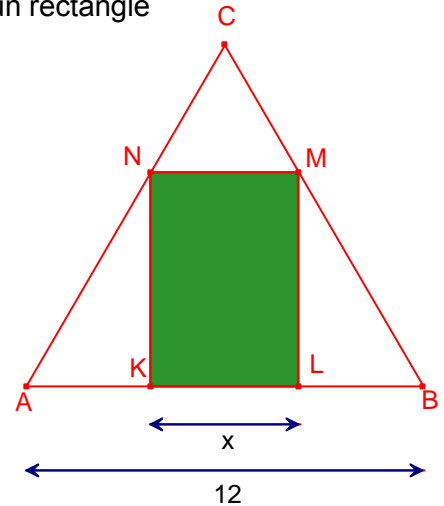
Àrea d'un rectangle inscrit en un triangle equilàter.

En un triangle equilàter $\triangle ABC$ de costat $\overline{AB} = 12$ s'ha inscrit un rectangle KLMN de costat $\overline{KL} = x$.

a) Si $x = 2$ calculeu l'àrea del rectangle, $S(2)$.

b) Ompliu la següent taula:

| x | S(x) |
|---|------|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |



c) Determineu l'àrea del rectangle KLMN en funció de $x = \overline{KL}$.

d) Descriviu les propietats de la funció.

e) Representeu la funció.

f) Per a quin valor x l'àrea del rectangle KLMN és màxima?. Quina és l'àrea màxima?.

Solució:

$$\overline{AK} = \overline{BL} = 6 - \frac{x}{2}.$$

$$\overline{MN} = \overline{CM} = \overline{CN}.$$

L'àrea del rectangle KLMN és igual a l'àrea del triangle equilàter $\triangle ABC$ menys la suma de les àrees de dos triangles equilàters de costats x i $12 - x$, respectivament.

$$S_{\text{KLMN}} = \frac{\sqrt{3}}{4} 12^2 - \left(\frac{\sqrt{3}}{4} x^2 + \frac{\sqrt{3}}{4} (12 - x)^2 \right).$$

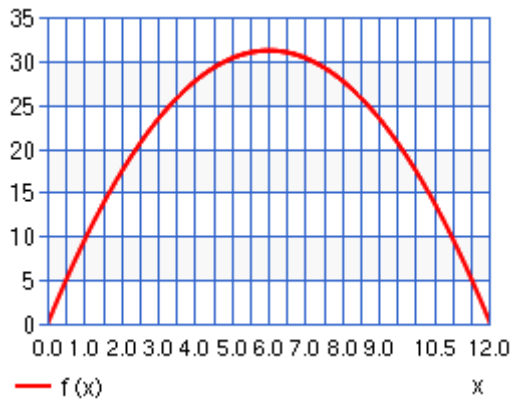
$$S(x) = \frac{\sqrt{3}}{2} (-x^2 + 12x), \quad x \in [0, 12].$$

Per calcular la taula utilitzarem el menú TAULA de la calculadora:

| | |
|--|-------------|
| $f(x) = \frac{\sqrt{3}}{2} (-x^2 + 12x)$ | Rango tabla |
| | Inic.: 0 |
| | Final: 12 |
| | Paso: 0.5 |

| <table border="1"> <thead> <tr> <th>x</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr><td>1</td><td>0</td></tr> <tr><td>2</td><td>4.9796</td></tr> <tr><td>3</td><td>9.5262</td></tr> <tr><td>4</td><td>13.639</td></tr> </tbody> </table> | x | $f(x)$ | 1 | 0 | 2 | 4.9796 | 3 | 9.5262 | 4 | 13.639 | <table border="1"> <thead> <tr> <th>x</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr><td>5</td><td>17.32</td></tr> <tr><td>6</td><td>20.568</td></tr> <tr><td>7</td><td>23.382</td></tr> <tr><td>8</td><td>25.764</td></tr> </tbody> </table> | x | $f(x)$ | 5 | 17.32 | 6 | 20.568 | 7 | 23.382 | 8 | 25.764 |
|---|--------|--------|----|--------|----|--------|----|--------|----|--------|--|-----|--------|----|--------|----|--------|----|--------|----|--------|
| x | $f(x)$ | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | | | | | | | | | | | | | | | | | | | | |
| 2 | 4.9796 | | | | | | | | | | | | | | | | | | | | |
| 3 | 9.5262 | | | | | | | | | | | | | | | | | | | | |
| 4 | 13.639 | | | | | | | | | | | | | | | | | | | | |
| x | $f(x)$ | | | | | | | | | | | | | | | | | | | | |
| 5 | 17.32 | | | | | | | | | | | | | | | | | | | | |
| 6 | 20.568 | | | | | | | | | | | | | | | | | | | | |
| 7 | 23.382 | | | | | | | | | | | | | | | | | | | | |
| 8 | 25.764 | | | | | | | | | | | | | | | | | | | | |
| 0 | 3.5 | | | | | | | | | | | | | | | | | | | | |
| <table border="1"> <thead> <tr> <th>x</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr><td>9</td><td>27.712</td></tr> <tr><td>10</td><td>29.228</td></tr> <tr><td>11</td><td>30.31</td></tr> <tr><td>12</td><td>30.96</td></tr> </tbody> </table> | x | $f(x)$ | 9 | 27.712 | 10 | 29.228 | 11 | 30.31 | 12 | 30.96 | <table border="1"> <thead> <tr> <th>x</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr><td>13</td><td>31.176</td></tr> <tr><td>14</td><td>30.96</td></tr> <tr><td>15</td><td>30.31</td></tr> <tr><td>16</td><td>29.228</td></tr> </tbody> </table> | x | $f(x)$ | 13 | 31.176 | 14 | 30.96 | 15 | 30.31 | 16 | 29.228 |
| x | $f(x)$ | | | | | | | | | | | | | | | | | | | | |
| 9 | 27.712 | | | | | | | | | | | | | | | | | | | | |
| 10 | 29.228 | | | | | | | | | | | | | | | | | | | | |
| 11 | 30.31 | | | | | | | | | | | | | | | | | | | | |
| 12 | 30.96 | | | | | | | | | | | | | | | | | | | | |
| x | $f(x)$ | | | | | | | | | | | | | | | | | | | | |
| 13 | 31.176 | | | | | | | | | | | | | | | | | | | | |
| 14 | 30.96 | | | | | | | | | | | | | | | | | | | | |
| 15 | 30.31 | | | | | | | | | | | | | | | | | | | | |
| 16 | 29.228 | | | | | | | | | | | | | | | | | | | | |
| 5.5 | 7.5 | | | | | | | | | | | | | | | | | | | | |
| <table border="1"> <thead> <tr> <th>x</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr><td>17</td><td>27.712</td></tr> <tr><td>18</td><td>25.764</td></tr> <tr><td>19</td><td>23.382</td></tr> <tr><td>20</td><td>20.568</td></tr> </tbody> </table> | x | $f(x)$ | 17 | 27.712 | 18 | 25.764 | 19 | 23.382 | 20 | 20.568 | <table border="1"> <thead> <tr> <th>x</th> <th>$f(x)$</th> </tr> </thead> <tbody> <tr><td>21</td><td>17.32</td></tr> <tr><td>22</td><td>13.639</td></tr> <tr><td>23</td><td>9.5262</td></tr> <tr><td>24</td><td>4.9796</td></tr> </tbody> </table> | x | $f(x)$ | 21 | 17.32 | 22 | 13.639 | 23 | 9.5262 | 24 | 4.9796 |
| x | $f(x)$ | | | | | | | | | | | | | | | | | | | | |
| 17 | 27.712 | | | | | | | | | | | | | | | | | | | | |
| 18 | 25.764 | | | | | | | | | | | | | | | | | | | | |
| 19 | 23.382 | | | | | | | | | | | | | | | | | | | | |
| 20 | 20.568 | | | | | | | | | | | | | | | | | | | | |
| x | $f(x)$ | | | | | | | | | | | | | | | | | | | | |
| 21 | 17.32 | | | | | | | | | | | | | | | | | | | | |
| 22 | 13.639 | | | | | | | | | | | | | | | | | | | | |
| 23 | 9.5262 | | | | | | | | | | | | | | | | | | | | |
| 24 | 4.9796 | | | | | | | | | | | | | | | | | | | | |
| 9.5 | 11.5 | | | | | | | | | | | | | | | | | | | | |

Utilitzarem el codi QR per representar la funció:



La funció és una paràbola convexa el màxim s'assoleix en el vèrtex és a dir, quan $x = 6$.

Per calcular-lo resoltem l'equació $S(x) = 0$ amb la calculadora que ens donarà el punts de tall i el vèrtex de la paràbola.

| | |
|--|--------------|
| ax^2+bx+c $- 0.866x^2+ 10.392x + \dots$ | 0 |
| $ax^2+bx+c=0$ $X_1 =$ | 12 |
| $ax^2+bx+c=0$ $X_2 =$ | 0 |
| Màx de $y=ax^2+bx+c$ $x =$ | 6 |
| Màx de $y=ax^2+bx+c$ $y =$ | $18\sqrt{3}$ |

El valor màxim s'assoleix quan $x = 6$ i l'àrea màxima és $S(6) = 18\sqrt{3}$.

Notem que \overline{MN} és paral·lela mitjana del triangle equilàter. Aleshores, \overline{KN} és igual a la meitat de l'altura del triangle equilàter.