

Siguen les esferes  $E_1 \equiv x^2 + y^2 + z^2 + 6x - 2y - 4z + 11 = 0$ ,  
 $E_2 \equiv (x - 1)^2 + (y + 2)^2 + (z - 2)^2 = 3^2$

- Demostreu que les dues esferes són tangents.
- Determineu l'equació del plànol tangent a les dues esferes.

Solució:

Completant quadrats:

$$E_1 \equiv (x + 3)^2 + (y - 1)^2 + (z - 2)^2 = -11 + 9 + 1 + 4 = 2^2$$

L'esfera té centre  $O_1(-3, 1, 2)$  i radi  $R_1 = 2$

L'esfera  $E_2 \equiv (x - 1)^2 + (y + 2)^2 + (z - 2)^2 = 3^2$  té centre  $O_2(1, -2, 2)$  i radi  $R_2 = 3$

Calculem la distància entre els centres:

$$O_1O_2 = \sqrt{4^2 + 3^2 + 0^2} = 5$$

Calculem la suma dels radis:

$$R_1 + R_2 = 2 + 3 = 5$$

Les esferes són tangents exteriors.

b)

$$E_2 \equiv x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$$

Calculem  $E_1 - E_2$

$$\pi \equiv 8x - 6y + 11 = 0, \text{ plànol tangent a les dues esferes.}$$

Obrim el *Menú Gráfico 3D*.

Definim i representem les dues esferes i el plànol.

The image shows three screenshots from a 3D graphics menu, likely from a TI-84 Plus calculator. Each screenshot shows a different type of object being defined.

- Top Left Screenshot:** Shows the definition of a sphere. The equation is  $(X-a)^2 + (Y-b)^2 + (Z-c)^2 = r^2$ . The parameters are set to  $a = -3$ ,  $b = 1$ ,  $c = 2$ , and  $r = 2$ . The sphere is visualized as a blue globe. The number '2' is shown in the bottom right corner.
- Top Right Screenshot:** Shows the definition of another sphere. The equation is  $(X-a)^2 + (Y-b)^2 + (Z-c)^2 = r^2$ . The parameters are set to  $a = 1$ ,  $b = -2$ ,  $c = 2$ , and  $r = 3$ . The sphere is visualized as a blue globe. The number '3' is shown in the bottom right corner.
- Bottom Screenshot:** Shows the definition of a plane. The equation is  $aX + bY + cZ + d = 0$ . The parameters are set to  $a = 8$ ,  $b = -6$ ,  $c = 0$ , and  $d = 11$ . The plane is visualized as a green grid. The number '11' is shown in the bottom right corner.

The bottom screenshot also shows a 3D coordinate system with X, Y, and Z axes. The X and Y axes range from -6 to 6, and the Z axis ranges from -6 to 6. The two spheres and the plane are plotted in this 3D space.