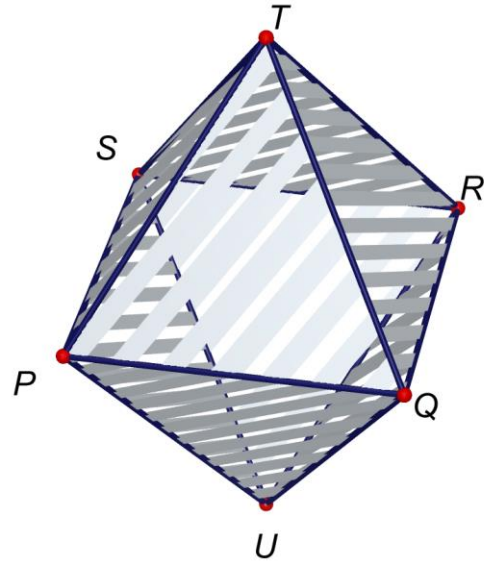


La cristallització d'un mineral és un octàedre amb els vèrtexs:

$P(1, -5, 2), Q(6, -3, 4), R(7, 1, 0), S(2, -1, -2)$
 $T(-4, 9, 10), U(12, -13, -8)$

- Proveu que els vectors $\overrightarrow{PR}, \overrightarrow{TU}$ es tallen perpendicularment per la meitat.
- Calculeu el volum de l'octàedre.
- Calculeu l'àrea de l'octàedre.



Solució:

a)

$$\overrightarrow{PQ} = (5, 2, 2)$$

$$\overrightarrow{SR} = (5, 2, 2)$$

Aleshores, PQRS és un paral·lelogram.

$$\overrightarrow{PR} = (6, 6, -2)$$

$$\overrightarrow{TU} = (16, -22, -18)$$

$$\overrightarrow{PR} \cdot \overrightarrow{TU} = 0$$

Aleshores, els vectors $\overrightarrow{PR}, \overrightarrow{TU}$ són perpendiculars.

Calculem el punt mig M de la diagonal \overrightarrow{PR} del paral·lelogram PQRS.

$$M(4, -2, 2)$$

$$\overrightarrow{TM} = (8, -11, -9)$$

$$\text{Aleshores, } \overrightarrow{TM} = \frac{1}{2} \overrightarrow{TU}$$

Per tant, els vectors $\overrightarrow{PR}, \overrightarrow{TU}$ es tallen perpendicularment en el punt $M(4, -2, 2)$

b)

$$\overrightarrow{PS} = (1, 4, -4), \overrightarrow{PT} = (-5, 14, 8)$$

El volum de l'octàedre és $V = 2 \cdot \frac{1}{3} \cdot |[\overrightarrow{PQ}, \overrightarrow{PS}, \overrightarrow{PT}]|$

Obrim el *Menú Ejec-Mat*

Definim la matriu formada pels tres vectors.

A	1	2	3
1	5	2	2
2	1	4	-4
3	-5	14	8

8

$\frac{2}{3} \text{Det Mat A} = \frac{1064}{3}$

El volum és

$$V_{\text{octàedre}} = \frac{1064}{3} u^3$$

b)

Definim els vectors $\overrightarrow{PQ} = (5, 2, 2)$, $\overrightarrow{PS} = (1, 4, -4)$, $\overrightarrow{TM} = (8, -11, -9)$

Calculator screen A: A 1x3 grid with values 5, 2, and 2. The grid is labeled with columns 1, 2, and 3. The value 2 in the third column is highlighted. Below the grid are buttons for ROW, COLUMN, and EDIT. The number 2 is displayed on the right side of the screen.

Calculator screen B: A 1x3 grid with values 1, 4, and -4. The grid is labeled with columns 1, 2, and 3. The value -4 in the third column is highlighted. Below the grid are buttons for ROW, COLUMN, and EDIT. The number -4 is displayed on the right side of the screen.

Calculator screen C: A 1x3 grid with values 8, -11, and -9. The grid is labeled with columns 1, 2, and 3. The value -9 in the third column is highlighted. Below the grid are buttons for ROW, COLUMN, and EDIT. The number -9 is displayed on the right side of the screen.

Calculem l'àrea del paral·lelogram PQRS, $S_{PQRS} = \|\overrightarrow{PQ} \times \overrightarrow{PS}\|$

Calculator screen: Shows the calculation of the cross product of vectors A and B. The result is a 1x3 grid with values [-16, 22, 18]. Below this, the norm of the resulting vector is calculated as $2\sqrt{266}$. The screen also shows the input $\text{CrossP}(\text{Vct A}, \text{Vct B})$ and $\text{Norm}(\text{Vct Ans})$.

$$S_{PQRS} = \|\overrightarrow{PQ} \times \overrightarrow{PS}\| = 2\sqrt{266}$$

Calculem $\|\overrightarrow{TM}\|$

Calculator screen: Shows the calculation of the norm of vector C. The result is $\sqrt{266}$. The screen also shows the input $\text{Norm}(\text{Vct C})$.

$$\|\overrightarrow{TM}\| = \sqrt{266}$$

El volum de l'octàedre és:

$$V = 2 \cdot \frac{1}{3} \|\overrightarrow{PQ} \times \overrightarrow{PS}\| \cdot \|\overrightarrow{TM}\|$$

Calculator display showing the calculation of the volume. The expression $\frac{4}{3} \times 266$ is entered, resulting in $\frac{1064}{3}$.

El volum és

$$V_{\text{octàedre}} = \frac{1064}{3} u^3$$

c)

Els triangles $\triangle PQT, \triangle RST, \triangle PQU, \triangle RSU$ són isòsceles i iguals.

Els triangles $\triangle PST, \triangle QRT, \triangle PSU, \triangle QRU$ són isòsceles i iguals.

L'àrea de l'octàedre és:

$$S_{\text{octàedre}} = 4 \left(\frac{1}{2} \cdot \|\overrightarrow{PQ} \times \overrightarrow{PT}\| + \frac{1}{2} \|\overrightarrow{PS} \times \overrightarrow{PT}\| \right)$$

$$S_{\text{octàedre}} = 2(\|\overrightarrow{PQ} \times \overrightarrow{PT}\| + \|\overrightarrow{PS} \times \overrightarrow{PT}\|)$$

Definim el vector $\overrightarrow{PT} = (-5, 14, 8)$

Calculator display showing a matrix D defined as:

D	1	2	3
1	1	-5	14
2			8
3			

The value 8 is shown in the bottom right corner of the matrix editor.

Calculem $\overrightarrow{PQ} \times \overrightarrow{PT}, \overrightarrow{PS} \times \overrightarrow{PT}$ i el seus mòduls



Calculator display showing the calculation of the cross product of vectors A and D, and its norm:

```
CrossP(Vct A, Vct D)
[-12 -50 80]
Norm(Vct Ans)
95.09994742
Ans→A
95.09994742
```

Calculator display showing the calculation of the cross product of vectors B and D, and its norm:

```
CrossP(Vct B, Vct D)
[88 12 34]
Norm(Vct Ans)
95.09994742
Ans→B
95.09994742
```

L'area és:

	Math	Rad	Norm1	d/c	a+bi
Norm(Vct	Ans)				
					95.09994742
Ans→B					95.09994742
2×(A+B)					380.3997897
<input type="checkbox"/>					
Vct	DotP(CrossP(Angle(UnitV(

$$S_{\text{octàedre}} = 2(\|\overrightarrow{PQ} \times \overrightarrow{PT}\| + \|\overrightarrow{PS} \times \overrightarrow{PT}\|) \approx 380.40$$

