

Siguen l'esfera $E \equiv x^2 + y^2 + z^2 = 9$ i els punts $A(3, 0, 6), B(3, 5, 1)$.
 Determineu els plànols tangents a l'esfera E que contenen els punts A, B .

Solució:

L'esfera $E \equiv x^2 + y^2 + z^2 = 9$ té centre l'origen de coordenades $O(0, 0, 0)$ i radi $r = 3$.

$$\overrightarrow{AB} = (0, 5, -5)$$

L'equació de la recta que passa pels punts A, B té equació:

$$r_{AB} \equiv (x, y, z) = (3, 0, 6) + \alpha(0, 1, -1)$$

L'equació implícita és:

$$r_{AB} \equiv \begin{cases} x - 3 = 0 \\ y + z - 6 = 0 \end{cases}$$

El feix de plànols que conté la recta r_{AB} és:

$$\pi_\mu \equiv (x - 3) + \mu(y + z - 6) = 0$$

$$\pi_\mu \equiv x + \mu y + \mu z - 3 - 6\mu = 0$$

El plànel π_μ ha de ser tangent a l'esfera aleshores:

$$d(O, \pi_\mu) = r = 3$$

$$\left| \frac{-3 - 3\mu}{\sqrt{1 + \mu^2 + \mu^2}} \right| = 3$$

Simplificant:

$$|1 + \mu| = \sqrt{1 + 2\mu^2}$$

Elevant al quadrat i simplificant:

$$2\mu^2 + 4\mu = 0$$

Resolent l'equació:

$$\mu = 0, -2$$

Els plànols que cerquem són:

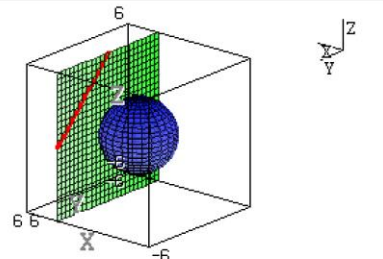
$$\pi_0 \equiv x - 3 = 0$$

$$\pi_{-2} \equiv x - 2y - 2z + 9 = 0$$

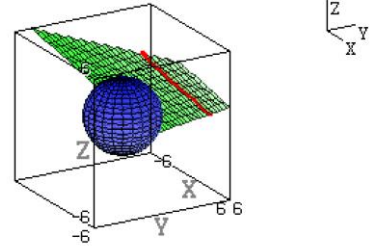
Obrim el *Menú Gráfico 3D*.

Definim i representem l'esfera E , la recta r_{AB} i el plànel π_0

The image shows two screenshots of a 3D graphics menu interface. The left screenshot displays the equation for a sphere: $(X-a)^2 + (Y-b)^2 + (Z-c)^2 = r^2$. The parameters are set as follows: $a=0$, $b=0$, $c=0$, and $r=3$. The right screenshot displays the equation for a line passing through two points: $\text{Recta pasa por 2 puntos}$. The points are $P1(3, 0, 6)$ and $P2(3, 5, 1)$. Both screenshots show a 3D coordinate system with axes X, Y, and Z. The left screenshot also includes a small 3D globe icon and a '3' in the bottom right corner. The right screenshot includes a small 3D line icon and a '1' in the bottom right corner. Both screenshots have a menu bar at the top with options: Math, Rad, Norm1, d/c, a+bi. The bottom of each screenshot has buttons: FACTOR, EXPAND, EDIT, SET, EXPRESS, VECTOR, P&V, POINTS, EDIT, SET.

<div style="display: flex; justify-content: space-between; border-bottom: 1px solid black; margin-bottom: 5px;"> Math Rad Norm1 d/c a+bi </div> <p style="margin: 0;">$aX+bY+cZ+d=0$</p> <div style="display: flex; justify-content: space-around; margin: 5px 0;"> a b c d </div> <div style="display: flex; justify-content: space-around; margin: 5px 0;"> [1 0 0 -3]</div> <div style="text-align: right; margin: 5px 0;">-3</div> <div style="display: flex; justify-content: space-between; border-top: 1px solid black; margin-top: 5px;"> EXPRESS VECTOR POINTS EDIT SET </div>	<div style="display: flex; justify-content: space-between; border-bottom: 1px solid black; margin-bottom: 5px;"> Math Rad Norm1 d/c a+bi </div> 
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Definim i representem l'esfera E , la recta r_{AB} i el plànol π_{-2}

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