

Considerem les següents matrius

$$A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 5 & 0 \end{pmatrix}$$

- Determineu la matriu X tal que $A^t X B^{-1} = C$, A^t és la transposada de A
- Calculeu el determinant de la matriu $B^{-1}(C^t C)B$, C^t és la transposada de C

Selectivitat, Andalusia 2015

Solució:

a)

Notem que la matriu X és d'ordre 2×3

$\det(A) = -3 \neq 0$, aleshores, la matriu A té inversa.

$\det(B) = 1 \neq 0$, aleshores, la matriu B té inversa.

$$A^t X B^{-1} = C$$

Multipliquem a l'esquerra per $(A^t)^{-1}$, i a la dreta per B

$$(A^t)^{-1} A^t X B^{-1} B = (A^t)^{-1} C B$$

$$I_2 X I_3 = (A^t)^{-1} C B, I_2 \text{ matriu unitat d'ordre 2, } I_3 \text{ matriu unitat d'ordre 3.}$$

$$X = (A^t)^{-1} C B$$

Obrim el *Menú Ejec-Mat*

Definim les tres matrius.

<p>A</p> <table border="1"> <tr><td>1</td><td>2</td></tr> <tr><td>1</td><td>-1</td></tr> <tr><td>2</td><td>-1</td></tr> </table> <p style="text-align: right;">- 1</p> <p>ROW-OP ROW COLUMN EDIT</p>	1	2	1	-1	2	-1	<p>B</p> <table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>-2</td><td>1</td><td>0</td></tr> <tr><td>3</td><td>2</td><td>1</td></tr> </table> <p style="text-align: right;">1</p> <p>ROW-OP ROW COLUMN EDIT</p>	1	2	3	1	0	0	-2	1	0	3	2	1
1	2																		
1	-1																		
2	-1																		
1	2	3																	
1	0	0																	
-2	1	0																	
3	2	1																	
<p>C</p> <table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>-1</td><td>5</td><td>0</td></tr> </table> <p style="text-align: right;">0</p> <p>ROW-OP ROW COLUMN EDIT</p>	1	2	3	1	0	0	-1	5	0										
1	2	3																	
1	0	0																	
-1	5	0																	

Calculem

$$X = (A^t)^{-1} C B$$

Math Rad Norm1 d/c | a+bi

(Trn Mat A)⁻¹ × Mat C × Mat B

-7	10/3	0
-3	5/3	0

Mat Mat→Lst Det Trn Augment ▶

$$X = \begin{pmatrix} -7 & \frac{7}{3} & 0 \\ -3 & \frac{5}{3} & 0 \end{pmatrix}$$

b)

Notem que $C^t C$ és una matriu d'ordre 3×3
 Aleshores, és pot efectuar $B^{-1}(C^t C)B$

$$\det(B^{-1}(C^t C)B) = \det(B^{-1}) \cdot \det(C^t C) \cdot \det B = \det(C^t C)$$

Calculem $\det(C^t C)$

The screenshot shows a TI-84 Plus calculator interface. At the top, there are mode buttons: Math, Rad, Norm1, d/c, and a+bi. The main display shows a 3x3 matrix with the following elements: the first row contains -7, 7/3, and 0; the second row contains -3, 5/3, and 0; the third row contains 0, 0, and 0. Below the matrix, the text 'Det (Trn Mat CxMat C)' is visible, followed by a right-pointing arrow and the number '0'. At the bottom, there is a menu bar with options: Mat, Mat→Lst, Det, Trn, and Augment, followed by a right-pointing arrow.

$$\det(B^{-1}(C^t C)B) = 0$$

Calculem $C^t C$

The screenshot shows a TI-84 Plus calculator interface. At the top, there are mode buttons: Math, Rad, Norm1, d/c, and a+bi. The main display shows a 3x3 matrix with the following elements: the first row contains 2, -5, and 0; the second row contains -5, 25, and 0; the third row contains 0, 0, and 0. Below the matrix, the text 'Det (Trn Mat CxMat C)' is visible, followed by a right-pointing arrow and the number '0'. At the bottom, there is a menu bar with options: Mat, Mat→Lst, Det, Trn, and Augment, followed by a right-pointing arrow.

$$C^t C = \begin{pmatrix} 2 & -5 & 0 \\ -5 & 25 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ el determinant és zero que ja la tercera fila de la matriu és zero.}$$