# PROBLEMES D'APLICACIÓ DE LA REGLA DE L'HÔPITAL.

a) $\lim\limits_{x\to 0} \dfrac{\ln(x^2+1)}{e^x - x - 1}$

b) $\lim\limits_{x\to 0} \dfrac{6\sin x - 6x + x^3}{6x^5}$

c) $\lim\limits_{x\to 0} \dfrac{\mathrm{tg}2x}{\mathrm{tg}3x}$

d) $\lim\limits_{x\to 0} \dfrac{1-\cos x^2}{x^2 \sin x^2}$

e) $\lim\limits_{x\to 0} \dfrac{\sin ax}{\sin bx}$

f) $\lim\limits_{x\to 0} \dfrac{\mathrm{tg}x - x}{x - \sin x}$

g) $\lim\limits_{x\to \infty} \dfrac{x^4 - x^2}{e^x + 1}$

h) $\lim\limits_{x\to 0} x^2 \ln x$

i) $\lim\limits_{x\to \frac{\pi}{2}} (\sin x - \cos x)^{\mathrm{tg}x}$

j) $\lim\limits_{x\to \infty} x \cdot \sin\dfrac{a}{x}$

k) $\lim\limits_{x\to 1} \left( \dfrac{x}{\ln x} - \dfrac{1}{\ln x} \right)$

l) $\lim\limits_{x\to 1} x^{\frac{1}{1-x}}$

m) $\lim\limits_{x\to 0} \dfrac{\sin x - x\cos x}{x(1-\cos x)}$

n) $\lim\limits_{x\to 0} \left( \dfrac{a^x + b^x}{2} \right)^{\frac{1}{x}}$

p) $\lim\limits_{x\to \frac{\pi}{2}} \dfrac{\ln(\sin x)}{(\pi - 2x)^2}$

q) $\lim\limits_{x\to 0} x^{7x}$

# Solucions

a) $$\lim_{x\to 0}\frac{\ln(x^2+1)}{e^x-x-1}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{\frac{2x}{x^2+1}}{e^x-1}=\lim_{x\to 0}\frac{2x}{(x^2+1)(e^x-1)}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{2}{2xe^x+(x^2+1)e^x}=2.$$

b) $$\lim_{x\to 0}\frac{6\sin x-6x+x^3}{6x^5}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{6\cos x-6+3x^2}{30x^4}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{-6\sin x+6x}{120x^3}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{-6\cos x+6}{360x^2}=$$

$$\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{6\sin x}{720x}=\frac{6}{720}\lim_{x\to 0}\frac{\sin x}{x}=\frac{6}{720}\cdot 1=\frac{1}{120}$$

c) $$\lim_{x\to 0}\frac{\operatorname{tg}2x}{\operatorname{tg}3x}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{\frac{2}{\cos^2 2x}}{\frac{3}{\cos^2 3x}}=\lim_{x\to 0}\frac{2\cos^2 3x}{3\cos^2 2x}=\frac{2}{3}$$

d) $$\lim_{x\to 0}\frac{1-\cos x^2}{x^2\sin x^2}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{\sin x^2\cdot 2x}{2x\sin x^2+2x^3\cos x^2}=\lim_{x\to 0}\frac{\sin x^2}{\sin x^2+x^2\cos x^2}=$$

$$\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{2x\cos x^2}{2x\cos x^2+2x\cos x^2-2x^3\sin x^2}=\lim_{x\to 0}\frac{\cos x^2}{2\cos x^2+x^2\sin x^2}=\frac{1}{2}$$

e) $$\lim_{x\to 0}\frac{\sin ax}{\sin bx}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{a\cos ax}{b\cos bx}=\frac{a}{b}$$

f) $$\lim_{x\to 0}\frac{\operatorname{tg}x-x}{x-\sin x}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{1+\operatorname{tg}^2x-1}{1-\cos x}=\lim_{x\to 0}\frac{\operatorname{tg}^2x}{1-\cos x}\underset{\frac{0}{0}H}{=}\lim_{x\to 0}\frac{2\operatorname{tg}x(1+\operatorname{tg}^2x)}{\sin x}=\lim_{x\to 0}\frac{2(1+\operatorname{tg}^2x)}{\cos x}=2$$

g) $$\lim_{x\to\infty}\frac{x^4-x^2}{e^x+1}\underset{\frac{\infty}{\infty}H}{=}\lim_{x\to\infty}\frac{4x^3-2x}{e^x}\underset{\frac{\infty}{\infty}H}{=}\lim_{x\to\infty}\frac{12x^2-2}{e^x}\underset{\frac{\infty}{\infty}H}{=}\lim_{x\to\infty}\frac{24x}{e^x}\underset{\frac{\infty}{\infty}H}{=}\lim_{x\to\infty}\frac{24}{e^x}=0$$

h) $$\lim_{x\to 0}x^2\ln x\underset{0\infty}{=}\lim_{x\to 0}\frac{\ln x}{\frac{1}{x^2}}\underset{\frac{\infty}{\infty}H}{=}\lim_{x\to 0}\frac{\frac{1}{x}}{\frac{-2}{x^3}}\lim_{x\to 0}\frac{x^2}{-2}=0$$

i) $$\lim_{x\to\frac{\pi}{2}}(\sin x-\cos x)^{\operatorname{tg}x}\underset{1^\infty}{=}\lambda$$

$$\ln\lambda=\lim_{x\to\frac{\pi}{2}}\ln\left((\sin x-\cos x)^{\operatorname{tg}x}\right)=\lim_{x\to\frac{\pi}{2}}\operatorname{tg}x(\ln(\sin x-\cos x))=\lim_{x\to\frac{\pi}{2}}\frac{\sin x\cdot\ln(\sin x-\cos x)}{\cos x}=$$

$$\underset{\frac{0}{0}H}{=}\lim_{x\to\frac{\pi}{2}}\frac{\cos x\cdot\ln(\sin x-\cos x)+\sin x\frac{\cos x+\sin x}{\sin x-\cos x}}{-\sin x}=-1,\qquad \ln\lambda=-1\Rightarrow\lambda=e^{-1}$$

j) $\lim\limits_{x\to\infty} x\cdot\sin\frac{a}{x} \underset{0\cdot\infty}{=} \lim\limits_{x\to\infty}\frac{\sin\frac{a}{x}}{\frac{1}{x}} \underset{\frac{0}{0}H}{=} \lim\limits_{x\to\infty}\frac{\cos\frac{a}{x}\cdot\frac{-a}{x^2}}{\frac{-1}{x^2}} = \lim\limits_{x\to\infty} a\cdot\cos\frac{a}{x} = a$

k) $\lim\limits_{x\to1}\left(\frac{x}{\ln x}-\frac{1}{\ln x}\right) \underset{\infty-\infty}{=} \lim\limits_{x\to1}\frac{x-1}{\ln x} \underset{\frac{0}{0}H}{=} \lim\limits_{x\to1}\frac{1}{\frac{1}{x}} = \lim\limits_{x\to1} x = 1$

l) $\lambda = \lim\limits_{x\to1} x^{\frac{1}{1-x}} = 1^{\infty}$

$\ln\lambda = \lim\limits_{x\to1}\ln\left(x^{\frac{1}{1-x}}\right) = \lim\limits_{x\to1}\frac{\ln x}{1-x} \underset{\frac{0}{0}H}{=} \lim\limits_{x\to1}\frac{\frac{1}{x}}{-1} = -1 \ln\lambda = -1 \Rightarrow \lambda = e^{-1}$

m) $\lim\limits_{x\to0}\frac{\sin x - x\cos x}{x(1-\cos x)} \underset{\frac{0}{0}H}{=} \lim\limits_{x\to0}\frac{\cos x-\cos x + x\sin x}{1-\cos x + x\sin x} = \lim\limits_{x\to0}\frac{x\sin x}{1-\cos x + x\sin x} =$

$\underset{\frac{0}{0}H}{=} \lim\limits_{x\to0}\frac{\sin x + x\cos x}{2\sin x + x\cos x} \underset{\frac{0}{0}H}{=} \lim\limits_{x\to0}\frac{2\cos x - x\sin x}{3\cos x - x\sin x} = \frac{2}{3}$

n) $\lambda = \lim\limits_{x\to0}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}} = 1^{\infty}$

$\ln\lambda = \lim\limits_{x\to0}\ln\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}} = \lim\limits_{x\to0}\frac{\ln\left(\frac{a^x+b^x}{2}\right)}{x} \underset{\frac{0}{0}H}{=} \lim\limits_{x\to0}\frac{\frac{2}{a^x+b^x}\cdot\frac{1}{2}(a^x\ln a + b^x\ln b)}{1} =$

$= \frac{1}{2}(\ln a + \ln b) = \ln(ab)^{1/2} \ln\lambda = \ln(ab)^{1/2} \Rightarrow \lambda = \sqrt{ab}$

p) $\lim\limits_{x\to\frac{\pi}{2}}\frac{\ln(\sin x)}{(\pi-2x)^2} \underset{\frac{0}{0}H}{=} \lim\limits_{x\to\frac{\pi}{2}}\frac{\frac{\cos x}{\sin x}}{2(\pi-2x)2} = \lim\limits_{x\to\frac{\pi}{2}}\frac{\cos x}{4(\pi-2x)\sin x} =$

$\underset{\frac{0}{0}H}{=} \lim\limits_{x\to\frac{\pi}{2}}\frac{-\sin x}{4(-2\sin x + (\pi-2x)\cos x)} = \frac{-1}{4(-2+0)} = \frac{1}{8}$

q) $\lambda = \lim\limits_{x\to0} x^{7x} = 0^0$

$\ln\lambda = \lim\limits_{x\to0}\ln\left(x^{7x}\right) = \lim\limits_{x\to0} 7x\ln x \underset{0\infty}{=} \lim\limits_{x\to0}\frac{\ln x}{\frac{1}{7x}} \underset{\frac{\infty}{\infty}H}{=} \lim\limits_{x\to0}\frac{\frac{1}{x}}{\frac{-1}{7x^2}} = \lim\limits_{x\to0} -7x = 0$

$\ln\lambda = 0 \Rightarrow \lambda = e^0 = 1$