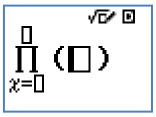
Dos problemes de logaritmes.

Problema 1:

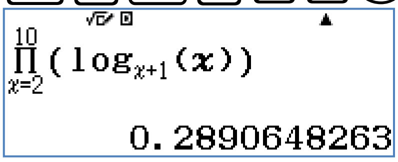
Comproveu que $\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot ..... \cdot \log_{11} 10 = \log_{11} 2$.

Demostreu la igualtat i feu la generalització:

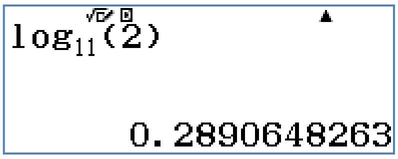
Solució:

Utilitzarem la funció productes finits [ALPHA] [x]  :

[ALPHA] [x] [log■□] [x] [+] [1] [▶] [x] [▶] [▶] [2] [▶] [1] [0] [=]



Calculem $\log_{11} 2$:



Els dos resultats són iguals.

Demostració:

Aplicant el canvi de bases de logaritmes:

$$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot ..... \cdot \log_{11} 10 = \frac{\log_{11} 2}{\log_{11} 3} \cdot \frac{\log_{11} 3}{\log_{11} 4} \cdot \frac{\log_{11} 4}{\log_{11} 5} \cdot ..... \frac{\log_{11} 9}{\log_{11} 10} \cdot \log_{11} 10 = \log_{11} 2$$

Generalització:

$\log_3 2 \cdot \log_4 3 \cdot \log_5 4 \cdot ..... \cdot \log_{n+1} n = \log_{n+1} 2, \quad n = 1, 2, 3, ...$

Problema 2

Comproveu que

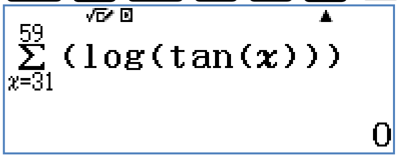
$\log(\text{tg } 31^{\circ}) + \log(\text{tg } 32^{\circ}) + \log(\text{tg } 33^{\circ}) + \ldots\ldots + \log(\text{tg } 59^{\circ}) = 0$

Solució:

La calculadora ha d'estar en mode sexagesimal d'angles.

Utilitzarem la funció sumes finites:

[SHIFT] [x] [SHIFT] [(−)] [tan] [x] [)] [)] [▶] [3] [1] [▶] [5] [9] [=]



Generalització:

$\log(\text{tg } 1^{\circ}) + \log(\text{tg } 2^{\circ}) + \log(\text{tg } 3^{\circ}) + \ldots\ldots + \log(\text{tg } 89^{\circ}) = 0$.

Demostració:

$\log(\text{tg } 1^{\circ}) + \log(\text{tg } 2^{\circ}) + \log(\text{tg } 3^{\circ}) + \ldots + \log(\text{tg } 89^{\circ}) =$

$= \log(\text{tg } 1^{\circ} \cdot \text{tg } 2^{\circ} \cdot \ldots \cdot \text{tg } 44^{\circ} \cdot \text{tg } 45^{\circ} \cdot \text{tg } 46^{\circ} \cdot \ldots \cdot \text{tg } 88^{\circ} \cdot \text{tg } 89^{\circ}) =$.

$= \log(\text{tg } 1^{\circ} \cdot \text{tg } 2^{\circ} \cdot \ldots \cdot \text{tg } 44^{\circ} \cdot \text{tg } 45^{\circ} \cdot \text{ctg } 44^{\circ} \cdot \ldots \cdot \text{ctg } 2^{\circ} \cdot \text{ctg } 1^{\circ}) = \log(1) = 0$