Problema 2

Siga ABCDEFG un heptàgon regular. Proveu que $\frac{1}{\overline{AB}} = \frac{1}{\overline{AC}} + \frac{1}{\overline{AD}}$.

Solució 1:

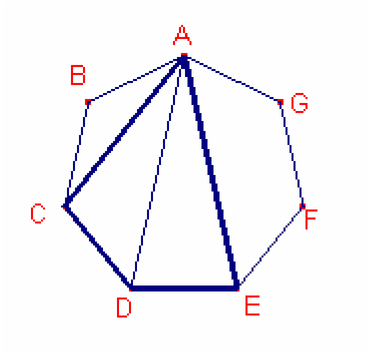
Siga $a = \overline{AB} = \overline{CD} = \overline{DE}, b = \overline{AC} = \overline{CE}, c = \overline{AD} = \overline{AE}$.

Aplicant el teorema de Tolomeu:

$ac + ab = bc$.

Dividint la igualtat per abc:

$\frac{1}{b} + \frac{1}{c} = \frac{1}{a}$.

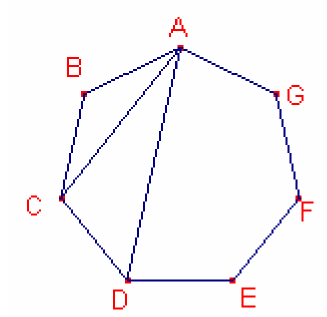


Solució 2:

Siga $a = \overline{AB}, b = \overline{AC}, c = \overline{AD}$.

Siga $\alpha = \angle BDA$. $7\alpha = 180^{o}$.

Aleshores, $\angle BAD = 2\alpha$, $\angle ABD = 4\alpha$.



Aplicant el teorema de Pitàgores al triangle $\overset{\Delta}{ABD}$:

$\frac{a}{\sin\alpha} = \frac{b}{\sin 2\alpha}$, aleshores, $b = \frac{\sin 2\alpha}{\sin\alpha} a$

$\frac{b}{\sin 2\alpha} = \frac{c}{\sin 4\alpha}$, aleshores, $c = b\frac{\sin 4\alpha}{\sin 2\alpha} = \frac{\sin 4\alpha}{\sin\alpha} a$.

$$\frac{1}{b} + \frac{1}{c} = \frac{\sin\alpha}{a \cdot \sin 2\alpha} + \frac{\sin\alpha}{a \sin 4\alpha} = \frac{1}{a}\left(\frac{\sin\alpha}{\sin 2\alpha} + \frac{\sin\alpha}{\sin 4\alpha}\right) = \frac{1}{a}\frac{\sin\alpha(\sin 4\alpha + \sin 2\alpha)}{\sin 2\alpha \cdot \sin 4\alpha} =$$

Notem que $4\alpha = 180^{o} – 4\alpha$, aleshores, $\sin 3\alpha = \sin 4\alpha$:

$$= \frac{1}{a}\frac{\sin\alpha \cdot 2 \cdot \sin 3\alpha \cdot \cos\alpha}{2\sin\alpha \cdot \cos\alpha \cdot \sin 3\alpha} = \frac{1}{a}$$