Problema

Siguen $z = \frac{-1+i\sqrt{3}}{4}$, $w = \frac{\sqrt{2}+i\sqrt{2}}{4}$

a) Escriviu z, w en la forma trigonomètrica $r(\cos\theta + i\sin\theta)$ amb $0 \leq \theta \leq \pi$

b) Proveu que $zw = \frac{1}{4}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

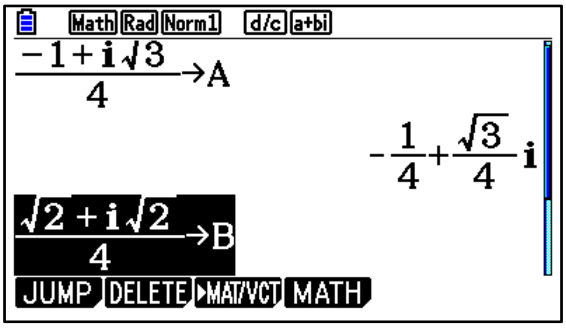
c) Calculeu zw en la forma binòmica $a + ib$ amb valors exactes.

Solució:

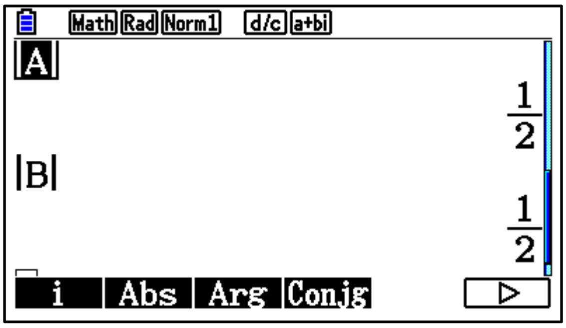
a)

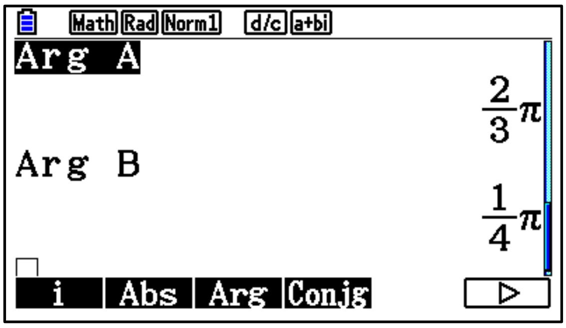
configurem la calculadora en forma binomial de complexos, $a + bi$.

Definim $\frac{-1+i\sqrt{3}}{4} \to A$, $\frac{\sqrt{2}+i\sqrt{2}}{4} \to B$



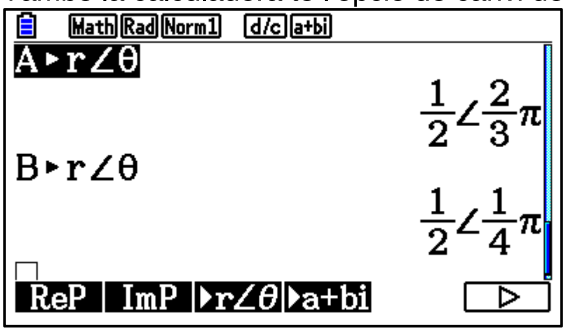
Calculem el mòdul i el argument de $z = \frac{-1+i\sqrt{3}}{4}$ i $w = \frac{\sqrt{2}+i\sqrt{2}}{4}$



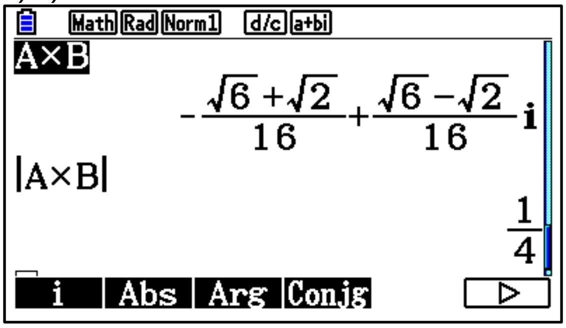


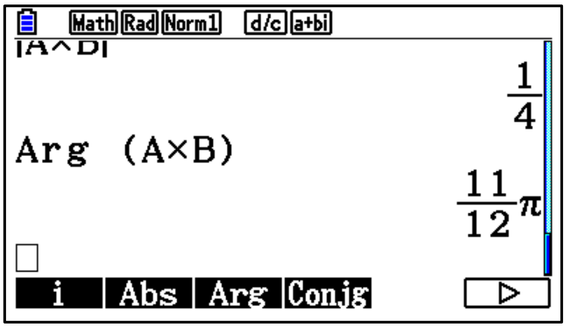
Aleshores, $z = \frac{-1+i\sqrt{3}}{4} = \frac{1}{2}\left(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right)$ $\quad w = \frac{\sqrt{2}+i\sqrt{2}}{4} = \frac{1}{2}\left(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi\right)$

També la calculadora té l'opció de canvi de format:



b) c)





En forma binòmica $zw = -\frac{\sqrt{6}+\sqrt{2}}{16} + \frac{\sqrt{6}-\sqrt{2}}{16} i$

Aleshores, $zw = \frac{1}{4}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

$$zw = |z|\cdot|w|(\cos(Arg\, z + Arg\, w) + i\ \sin(Arg\, z + Arg\, w)) = \frac{1}{4}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$$

També la calculadora té l’opció de canvi de format: